



International Baccalaureate<sup>™</sup>  
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# Mathematics SL formula booklet

For use during the course and in the examinations

First examinations 2014



# Contents

<b>Prior learning</b>	<b>2</b>
<b>Topics</b>	<b>3</b>
<hr/>	
Topic 1—Algebra	3
Topic 2—Functions and equations	4
Topic 3—Circular functions and trigonometry	4
Topic 4—Vectors	5
Topic 5—Statistics and probability	5
Topic 6—Calculus	6

## Prior learning

Area of a parallelogram	$A = b \times h$
Area of a triangle	$A = \frac{1}{2}(b \times h)$
Area of a trapezium	$A = \frac{1}{2}(a + b)h$
Area of a circle	$A = \pi r^2$
Circumference of a circle	$C = 2\pi r$
Volume of a pyramid	$V = \frac{1}{3}(\text{area of base} \times \text{vertical height})$
Volume of a cuboid (rectangular prism)	$V = l \times w \times h$
Volume of a cylinder	$V = \pi r^2 h$
Area of the curved surface of a cylinder	$A = 2\pi r h$
Volume of a sphere	$V = \frac{4}{3}\pi r^3$
Volume of a cone	$V = \frac{1}{3}\pi r^2 h$
Distance between two points $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
Coordinates of the midpoint of a line segment with endpoints $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$	$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$

## Topic I—Algebra

<p><b>1.1</b></p>	<p>The <math>n^{\text{th}}</math> term of an arithmetic sequence</p> <p>The sum of <math>n</math> terms of an arithmetic sequence</p> <p>The <math>n^{\text{th}}</math> term of a geometric sequence</p> <p>The sum of <math>n</math> terms of a finite geometric sequence</p> <p>The sum of an infinite geometric sequence</p>	$u_n = u_1 + (n-1)d$ $S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$ $u_n = u_1 r^{n-1}$ $S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}, \quad r \neq 1$ $S_\infty = \frac{u_1}{1 - r}, \quad  r  < 1$
<p><b>1.2</b></p>	<p>Exponents and logarithms</p> <p>Laws of logarithms</p> <p>Change of base</p>	$a^x = b \Leftrightarrow x = \log_a b$ $\log_c a + \log_c b = \log_c ab$ $\log_c a - \log_c b = \log_c \frac{a}{b}$ $\log_c a^r = r \log_c a$ $\log_b a = \frac{\log_c a}{\log_c b}$
<p><b>1.3</b></p>	<p>Binomial coefficient</p> <p>Binomial theorem</p>	$\binom{n}{r} = \frac{n!}{r!(n-r)!}$ $(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$

## Topic 2—Functions and equations

<b>2.4</b>	Axis of symmetry of graph of a quadratic function	$f(x) = ax^2 + bx + c \Rightarrow$ axis of symmetry $x = -\frac{b}{2a}$
<b>2.6</b>	Relationships between logarithmic and exponential functions	$a^x = e^{x \ln a}$ $\log_a a^x = x = a^{\log_a x}$
<b>2.7</b>	Solutions of a quadratic equation  Discriminant	$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, a \neq 0$  $\Delta = b^2 - 4ac$

## Topic 3—Circular functions and trigonometry

<b>3.1</b>	Length of an arc  Area of a sector	$l = \theta r$  $A = \frac{1}{2} \theta r^2$
<b>3.2</b>	Trigonometric identity	$\tan \theta = \frac{\sin \theta}{\cos \theta}$
<b>3.3</b>	Pythagorean identity  Double angle formulae	$\cos^2 \theta + \sin^2 \theta = 1$  $\sin 2\theta = 2 \sin \theta \cos \theta$  $\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$
<b>3.6</b>	Cosine rule  Sine rule  Area of a triangle	$c^2 = a^2 + b^2 - 2ab \cos C; \cos C = \frac{a^2 + b^2 - c^2}{2ab}$  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$  $A = \frac{1}{2} ab \sin C$

## Topic 4—Vectors

<b>4.1</b>	Magnitude of a vector	$ \mathbf{v}  = \sqrt{v_1^2 + v_2^2 + v_3^2}$
<b>4.2</b>	Scalar product	$\mathbf{v} \cdot \mathbf{w} =  \mathbf{v}  \mathbf{w} \cos\theta$ $\mathbf{v} \cdot \mathbf{w} = v_1w_1 + v_2w_2 + v_3w_3$
	Angle between two vectors	$\cos\theta = \frac{\mathbf{v} \cdot \mathbf{w}}{ \mathbf{v}  \mathbf{w} }$
<b>4.3</b>	Vector equation of a line	$\mathbf{r} = \mathbf{a} + t\mathbf{b}$

## Topic 5—Statistics and probability

<b>5.2</b>	Mean of a set of data	$\bar{x} = \frac{\sum_{i=1}^n f_i x_i}{\sum_{i=1}^n f_i}$
<b>5.5</b>	Probability of an event $A$	$P(A) = \frac{n(A)}{n(U)}$
	Complementary events	$P(A) + P(A') = 1$
<b>5.6</b>	Combined events	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
	Mutually exclusive events	$P(A \cup B) = P(A) + P(B)$
	Conditional probability	$P(A \cap B) = P(A)P(B A)$
	Independent events	$P(A \cap B) = P(A)P(B)$
<b>5.7</b>	Expected value of a discrete random variable $X$	$E(X) = \mu = \sum_x x P(X = x)$
<b>5.8</b>	Binomial distribution	$X \sim B(n, p) \Rightarrow P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}, r = 0, 1, \dots, n$
	Mean	$E(X) = np$
	Variance	$\text{Var}(X) = np(1-p)$
<b>5.9</b>	Standardized normal variable	$z = \frac{x - \mu}{\sigma}$

## Topic 6—Calculus

<b>6.1</b>	Derivative of $f(x)$	$y = f(x) \Rightarrow \frac{dy}{dx} = f'(x) = \lim_{h \rightarrow 0} \left( \frac{f(x+h) - f(x)}{h} \right)$
<b>6.2</b>	Derivative of $x^n$	$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$
	Derivative of $\sin x$	$f(x) = \sin x \Rightarrow f'(x) = \cos x$
	Derivative of $\cos x$	$f(x) = \cos x \Rightarrow f'(x) = -\sin x$
	Derivative of $\tan x$	$f(x) = \tan x \Rightarrow f'(x) = \frac{1}{\cos^2 x}$
	Derivative of $e^x$	$f(x) = e^x \Rightarrow f'(x) = e^x$
	Derivative of $\ln x$	$f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$
	Chain rule	$y = g(u), u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
	Product rule	$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$
	Quotient rule	$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
<b>6.4</b>	Standard integrals	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$ $\int \frac{1}{x} dx = \ln x + C, \quad x > 0$ $\int \sin x dx = -\cos x + C$ $\int \cos x dx = \sin x + C$ $\int e^x dx = e^x + C$
<b>6.5</b>	Area under a curve between $x = a$ and $x = b$	$A = \int_a^b y dx$
	Volume of revolution about the $x$ -axis from $x = a$ to $x = b$	$V = \int_a^b \pi y^2 dx$
<b>6.6</b>	Total distance travelled from $t_1$ to $t_2$	$\text{distance} = \int_{t_1}^{t_2}  v(t)  dt$



